

Deep Reinforcement Learning

University of Cambridge

Overview

- 1. Lecture (approx. 1 hour)
 - 1. Review
 - 2. Finish up MDPs
 - 3. Agents/policies
 - 4. Derive and define Q learning
- 2. Discussion/questions (approx. 30 mins)

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Review

Last time we talked about MDPs

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Designed state and action spaces for Mario

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Let's briefly review the MDP

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 $\gamma \in [0,1]$ is a discount factor that we will explain later

 $T: S \times A \rightarrow \Delta S$ The state transition function.

 $T: S \times A \to \Delta S \text{ The state transition function.}$ $T\left(\underbrace{\begin{bmatrix} x_1 & y_1 & x_2 & y_2 & \dots \end{bmatrix}}_{\text{state}}, \underbrace{\begin{bmatrix} F_x & F_y & i \end{bmatrix}}_{\text{action}}\right) = \underbrace{\Delta \begin{bmatrix} x_1 & y_1 & x_2 & y_2 & \dots \end{bmatrix}}_{\text{new state}}$

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This is a **Markov** decision process because transition dynamics are **conditionally independent** of past states and actions

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If conditional independence is violated, the process is **not** Markov

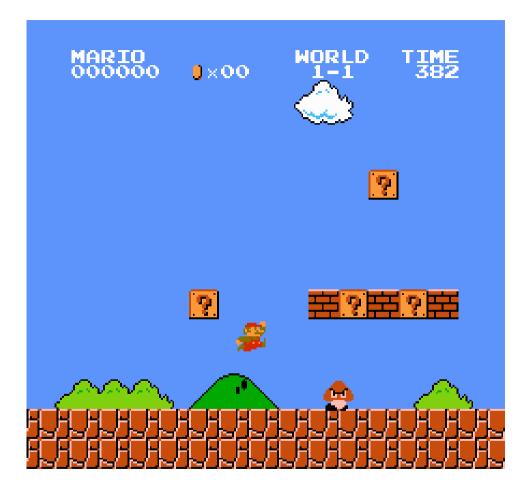
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$$T(s_t, a_t) \neq T(s_t, a_t \mid s_{t-1}) \Longleftarrow \text{Not Markov!}$$



• State Space (S)?



- State Space (S)?
 - The position and velocity (x,y,\dot{x},\dot{y}) of Mario and Goombas
 - The score
 - Number of coins collected
 - The time remaining
 - Which question blocks have been opened
 - Which goombas have been squished



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$$S = \left\{ \mathbb{R}^4, \mathbb{R}^4, ..., \mathbb{Z}_+, \mathbb{Z}_+, \mathbb{Z}_+, \left\{ 0, 1 \right\}^k \right\}$$



• State Space (S)?



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2x256x240x3 pixels.

E.g.
$$\begin{pmatrix} \binom{255}{0} & \binom{170}{10} & \dots \\ 50 & \binom{10}{50} & \binom{255}{0} & \binom{170}{10} & \dots \\ \binom{10}{100} & \binom{200}{200} & \dots \\ \vdots & \ddots \end{pmatrix}, \begin{pmatrix} \binom{255}{0} & \binom{170}{10} & \dots \\ \binom{10}{50} & \binom{200}{200} & \dots \\ \frac{235}{35} & \binom{200}{35} & \dots \\ \vdots & \ddots \end{pmatrix}$$

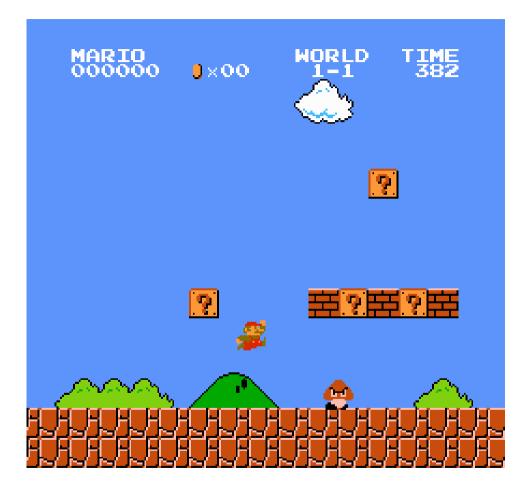


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 $S = \mathbb{Z}^{2 \times 256 \times 240 \times 3}_{< 255}$



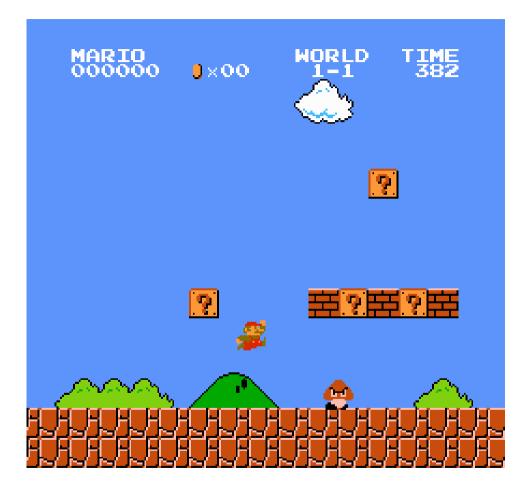
• Action Space (A)?



- Action Space (A)?
 - Acceleration of Mario \ddot{x}



- Action Space (A)?
 - Acceleration of Mario \ddot{x}
 - But when playing Mario, we cannot explicitly set \ddot{x}



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 - The Nintendo controller has A,

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 A = {0,1}⁶



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 - The Nintendo controller has A,
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$$A = \{0, 1\}^{6} \left\{ \underbrace{\{0, 1, 2, 3, 4\}}_{\emptyset, \text{direction}}, \underbrace{\{0, 1, 2, 3\}}_{\emptyset, \text{a,b,a+b}} \right\}$$

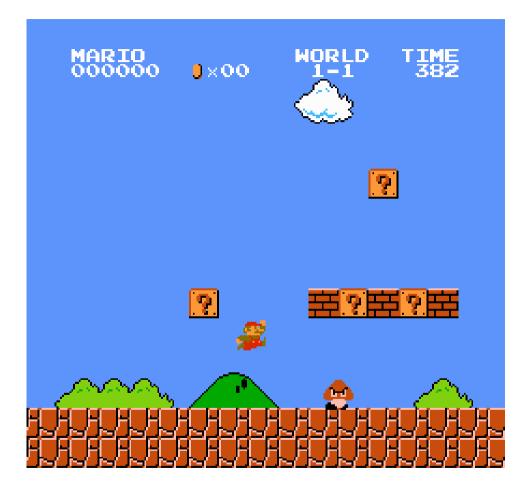
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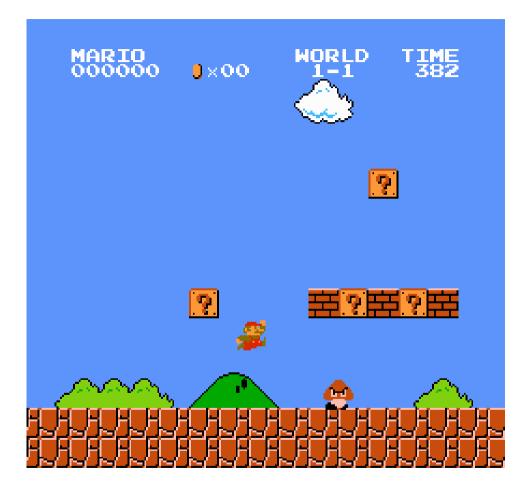
• Transition Function (T)?



- Transition Function (T)?
 - T(pixel_state, right)



- Transition Function (T)?
 - $T(pixel_state, right)$
 - Move the Mario pixels right, unless a wall
 - Difficult to write down
 - Deterministic



• Transition Function (T)?



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 - $T(\text{pos_vel_state}, \text{acc. right})$



- Transition Function (T)?
 - $T(\text{pos_vel_state}, \text{acc. right})$
 - Changes Mario's (x, y, \dot{x}, \dot{y}) in game memory
 - Human understandable, easier to implement for game developers

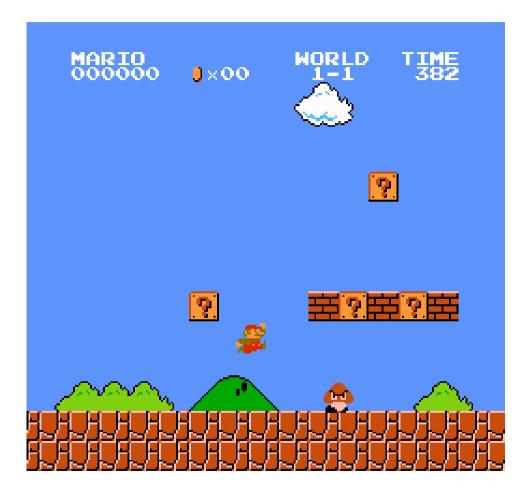


Question: In Mario, a single image frame is not a Markov state. How come?



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Answer: Cannot measure velocity.



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Answer: If we don't have it, Markov property is violated

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Not conditionally independent! $T(s_t, a_t \mid s_{t-1}, a_{t-1}, ..., s_0, a_0) \neq T(s_t, a_t)$



• **Reward** (*R*)?



- **Reward** (*R*)?
 - 1 for beating the level and 0 otherwise



- **Reward** (*R*)?
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 - Total score



- **Reward** (*R*)?
 - 1 for beating the level and 0 otherwise
 - Total score
 - 1 for beating the level +
 0.01 · score

- *S***√**
- A <
- *T* <
- $R\checkmark$
- γ?

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Note that we care about all future rewards, not just the current reward!

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$$\begin{split} G = \sum_{t=0}^\infty \gamma^t r_t = r_0 + \gamma r_1 + \gamma^2 r_2 + \dots \\ 0 \leq \gamma \leq 1 \end{split}$$

$$G = \sum_{t=0}^{\infty} \gamma^t r_t = r_0 + \gamma r_1 + \gamma^2 r_2 + \dots$$
$$0 \le \gamma \le 1$$

With a reward of 1 at each timestep and $\gamma = 0.9$

$$G = \sum_{t=0}^{\infty} \gamma^t r_t = 1 + 0.9 + 0.81 + \ldots = \frac{1}{1 - \gamma} = 10$$

Exercise: Reinforcement learning also describes human and animal behaviors. How can you describe your behavior using reinforcement learning?

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This happens internally when I decide to go to the pub after work

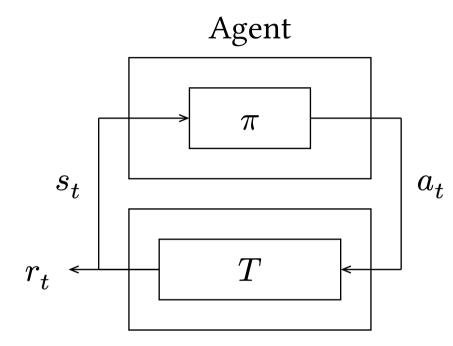


Agents and Policies

Deep Reinforcement Learning

University of Cambridge

Reinforcement Learning

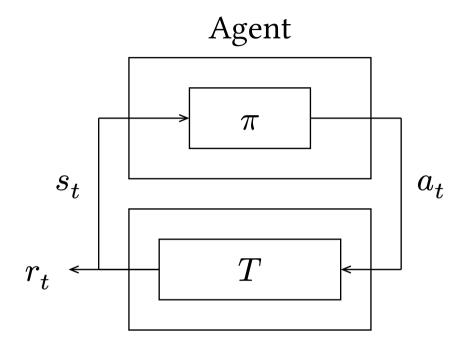


• We have defined the environment

Environment

 s_t : state, a_t : action, r_t : reward, π : policy, T: transition fn

Reinforcement Learning



- We have defined the environment
- Now let us define the agent

Environment

 s_t : state, a_t : action, r_t : reward, π : policy, T: transition fn

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 $a_t \sim \pi(s_t)$

 $\pi(a_t \mid s_t)$

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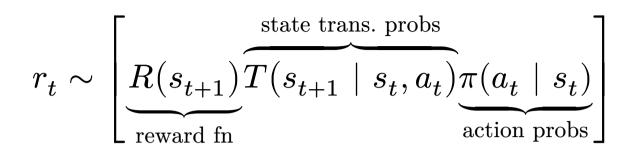
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$$r_t \sim \left[\underbrace{\frac{R(s_{t+1})}{T(s_{t+1} \mid s_t, a_t)}}_{\text{reward fn}} \overline{T(s_{t+1} \mid s_t, a_t)} \underbrace{\pi(a_t \mid s_t)}_{\text{action probs}}\right]$$



The **expectation** turns that distribution into a single number. This tells us what reward to expect "on average"

$$\mathbb{E}[r_t] = \int_{s_{t+1}} \int_A \underbrace{\frac{R(s_{t+1})}_{\text{reward fn}} \overline{T(s_{t+1} \mid s_t, a_t)}}_{\text{reward fn}} \underbrace{\frac{\pi(a_t \mid s_t)}{\pi(a_t \mid s_t)}}_{\text{action probs}}$$

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Now our policy is optimal with respect to all the uncertainty present!

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Popular algorithms:

• Deep Q Networks (DQN)

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- Advantage Weighted Regression (AWR)

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- Asynchronous Actor Critic (A2C)

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The Plan:

1. Derive the value function V

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- 1. Derive the value function V
- 2. Derive Q function from V

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- 1. Derive the value function V
- 2. Derive Q function from V
- 3. Figure out a behavior policy using ${\cal Q}$
- 4. Learn to train Q

Recall the discounted return of a specific policy π

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In English: At each timestep, we take an action $a_t \sim \pi(s_t)$

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In English: At each timestep, we take an action $a_t \sim \pi(s_t)$ follow the state transition function $s_{t+1} \sim T(s_t, a_t)$

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In English: At each timestep, we take an action $a_t \sim \pi(s_t)$

follow the state transition function $s_{t+1} \sim T(s_t, a_t)$

and get a reward $r_t = R(s_{t+1})$

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Rather than start the return from a given timestep, what if we defined it from a given state?

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We call this the Value Function (V_{π}) $V_{\pi}: S \to \mathbb{R}$

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Measures the value of a state (how good is it to be in this state?), for a given policy π

Step 2: Deriving Q

The Plan:

- 1. Derive the value function ${\cal V}$
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When *V* depends on a specific action, we call it the **Q** function:

$$S \times A \to \mathbb{R}$$

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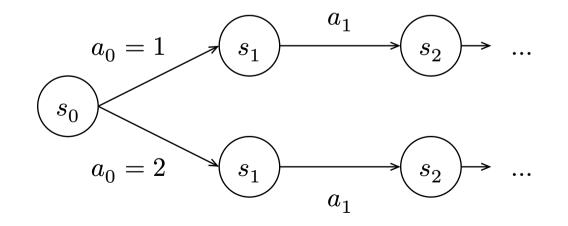
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 a_0 affects your next state s_1 , which affects the future



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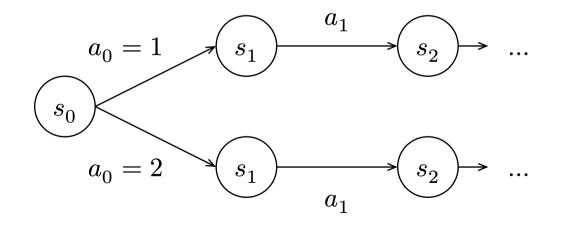
Q function gives you a number denoting how much better your life will be for attending Cambridge (based on your behavior π). Takes into account reward (based on income, friend group, experiences, etc).

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$$\begin{split} Q(s_0, \text{Cambridge}) &= f(\text{friends} + \text{experiences} + \text{income}) = 1200\\ Q(s_0, \text{Oxford}) &= f(\text{friends} + \text{experiences} + \text{income}) = 900 \end{split}$$

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Richard Bellman proved that a greedy policy is optimal (see the Bellman Equation)

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In English: Just take things one step at a time. Compute Q value for all possible actions and pick the action with the biggest Q value. Repeat at each timestep.

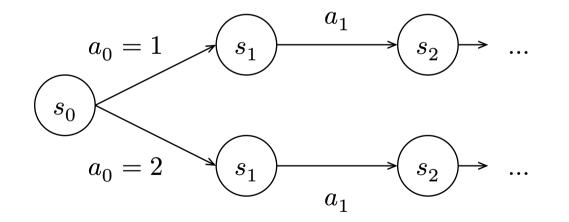
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Sidenote: OpenAI's leaked AI breakthrough named Q_* is likely related to this!

Step 4: Train Q

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After infinite time, we will have one datapoint for training. Can we get rid of the infinite sum?

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It is the Q function starting at s_1 , a recursive formulation!

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With the infinite sum gone, this is much easier to compute

Summary

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We defined the Q function training objective

$$\min_{\boldsymbol{\theta}} \left(Q(s, a, \boldsymbol{\theta}) - \left(r + \gamma \cdot \operatorname*{argmax}_{\{a' \in A\}} Q(s', a', \boldsymbol{\theta}) \right) \right)^2$$

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Next Time: We will focus on a practical implementation of Deep Q Learning

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 - Name and 2-3 sentences why they chose this module

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