Deep Q Networks

Deep Reinforcement Learning

University of Cambridge

Agenda

- Review

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- State of the field

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- Implement Deep Q Networks (DQN) (Mnih et al.)

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- Finished up MDPs

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The **discounted return (G)**

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With a reward of 1 at each timestep and $\gamma = 0.9$

$$G = \sum_{t=0}^{\infty} \gamma^t r_t = 1 + 0.9 + 0.81 + ... = \frac{1}{1 - \gamma} = 10$$

Review

The **expected discounted return (G_π)**

$$G_\pi = \mathbb{E}\left[\sum_{t=0}^{\infty} \gamma^t r_t \mid a_t \sim \pi(s_t)\right]$$

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$$r_t \sim \left[\underbrace{R(s_{t+1})}_{\text{reward fn}} \overbrace{T(s_{t+1} \mid s_t, a_t)}^{\text{state trans. probs}} \underbrace{\pi(a_t \mid s_t)}_{\text{action probs}}\right]$$

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The **expectation** turns that distribution into a single number. This tells us what reward to expect "on average"

$$\mathbb{E}[r_t] = \int_{s_{t+1}} \int_A \underbrace{R(s_{t+1})}_{\text{reward fn}} \overbrace{T(s_{t+1} \mid s_t, a_t)}^{\text{state trans. probs}} \underbrace{\pi(a_t \mid s_t)}_{\text{action probs}}$$

Review

$$G_\pi = \mathbb{E}\left[\sum_{t=0}^{\infty} \gamma^t r_t \mid a_t \sim \pi(s_t)\right]$$

With the expected discounted return, we can define the optimal policy

$$\pi_* = \max_{\pi} \ \mathbb{E}\left[\sum_{t=0}^{\infty} \gamma^t r_t \mid a_t \sim \pi(s_t)\right]$$

Review

Review:

- Finished up MDPs
- The return and optimal policies
- **Deep Q learning**

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The Plan:

1. Derive the value function V
2. Derive Q function from V
3. Figure out a behavior policy using Q
4. Learn to train Q

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$$V_\pi(s_0) = \mathbb{E}\left[\sum_{t=0}^{\infty} \gamma^t r_t \mid a_t \sim \pi(s_t)\right]$$

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Difference between G_π and V_π is dependence on s_0

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2. **Derive Q function from V**
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Factor out first term from the return to introduce a dependence on a_0

$$V_\pi(s_0, a_0) = \mathbb{E}[r_0 \mid a_0] + \mathbb{E}\left[\sum_{t=1}^{\infty} \gamma^t r_t \mid a_t \sim \pi(s_t)\right]$$

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When V depends on a specific action, we call it the **Q function**:

$$S \times A \to \mathbb{R}$$

$$Q_\pi(s_0, a_0) = \mathbb{E}[r_0 \mid a_0] + \mathbb{E}\left[\sum_{t=1}^{\infty} \gamma^t r_t \mid a_t \sim \pi(s_t)\right]$$

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$$\pi_*(s) = \underset{a \in A}{\operatorname{argmax}}\, Q_*(s, a)$$

In English: Compute Q value for all possible actions and pick the action with the biggest Q value. Repeat at each timestep.

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$$Q(s, a) = r + \gamma \cdot \max_{\{a' \in A\}} Q(s', a')$$

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$$Q(s, a, \theta) - \left(r + \gamma \cdot \max_{\{a' \in A\}} Q(s', a', \theta) \right) = 0$$

Training objective

$$\min_{\theta} \left(Q(s, a, \theta) - \left(r + \gamma \cdot \max_{\{a' \in A\}} Q(s', a', \theta) \right) \right)^2$$

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Since its inception, Deep RL has added "patches" to combine theory with deep networks to obtain better and better results

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dataset = load_dataset()
model = nn.Module(dataset.x.size, dataset.y.size)
theta = model.init(seed=0) # Functional

for update in range(num_updates):
  train_data = dataset.sample()
  theta = train(model, theta, train_data) # Functional
  metrics = evaluate(model, theta, dataset.val_set)
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  collected_data = collect_training_data(env, pi)
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The Environment

Main differences with the DL train loop

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The Environment

Today: Go through the loop line by line to implement DQN

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The Environment

Instead of loading a static dataset, we collect data from an environment

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The Environment

The Plan:

1. Terminal states
2. The Gymnasium interface

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1. **Terminal states**
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Terminal States

Question: How can we represent a Mario Bros Game Over screen in an MDP?

Terminal States

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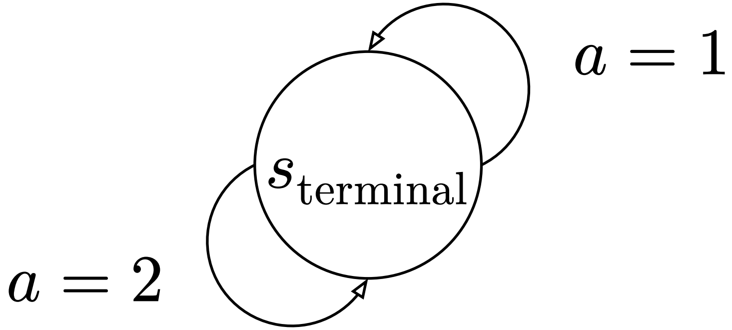
Answer: We enter a **terminal state** that we cannot leave



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Recall the discounted return

$$G = \sum_{t=0}^{\infty} \gamma^t r_t$$

After entering the terminal state at $t = n$ all future rewards are zero. We can write the discounted return as

$$G = \sum_{t=0}^{\infty} \gamma^t r_t \cdot (t \leq n) = \sum_{t=0}^{n} \gamma^t r_t$$

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$$\begin{pmatrix} s_0 & s_1 & s_2 & \dots & s_n \\ d_0 = 0 & d_1 = 0 & d_2 = 0 & \dots & d_n = 1 \end{pmatrix}$$

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We call the states from the initial to terminal state an **episode**

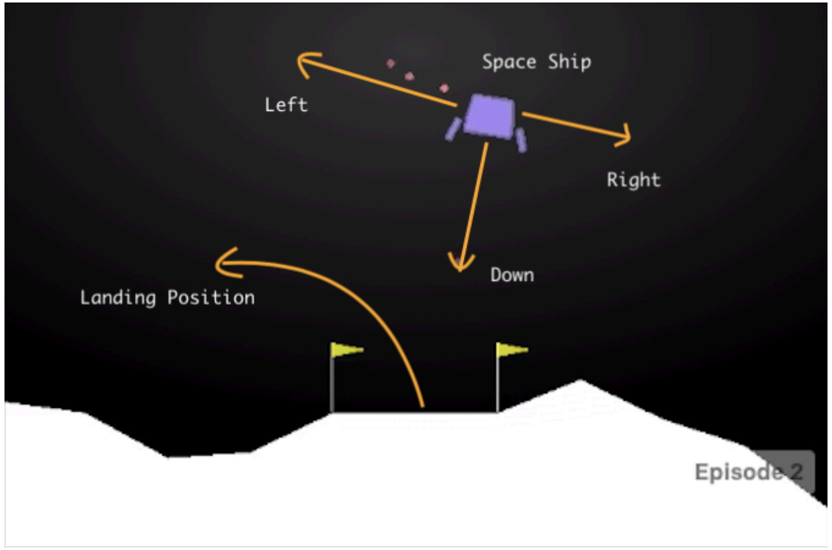
The Environment

The Plan:

1. Terminal states
2. **The Gymnasium interface**

Environment Libraries

The `gymnasium` library contains many popular test environments



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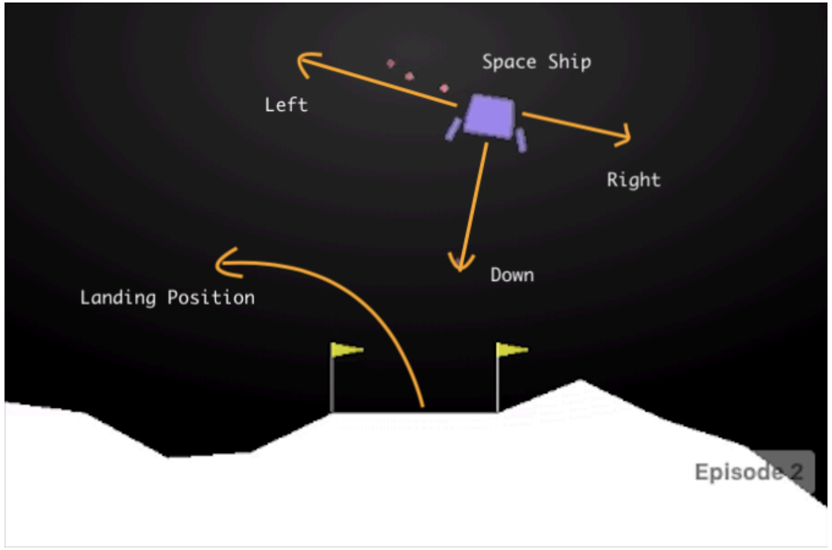

`gymnasium` also defines the standard environment interface

The Gym Interface

Launching environments is very easy

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Observations are states that are not guaranteed to be Markov. For LunarLander, they are Markov.

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Step the environment in time by feeding an action (transition function T)

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next_state, reward, terminated, truncated, _ = env.step(action)
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$$d = \text{terminated} \lor \text{truncated}$$

The Gym Interface

You can write your own environments using `gymnasium`

```
class RoboParking(gymnasium.Env):
  observation_space = spaces.Box(
    # x, y, xdot, ydot
    low=(0, 0, -1, -1),
    high=(4, 4, 1, 1),
    dtype=np.float32
  )
  # left, right, forward, backward
  action_space = spaces.Discrete(4)

  def R(self, pos): # R(s')
    # Our goal is 0,0,0,0
    return norm(sensor.state())
```

```
def T(self, action):
  # We don't know the true T
  wheels.apply_torque(action)
  next_state = sensor.state()
  return next_state

def step(self, action):
  next_state = T(action) # s'
  return (
    next_state,
    R(next_state), # reward R(s')
    norm(next_state) < 0.01 # d
    False, {} # trunc, extra info
  )
```

We discussed the environment

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Next, let us define the deep Q function

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Inefficient: $|A| = 100$ means 100 forward passes for each timestep

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Compute the Q value for all actions in a single forward pass

$$\begin{aligned} Q(s, \theta) = & \; Q(s, a = 1, \theta), \\ & \; Q(s, a = 2, \theta), \\ & \; \vdots \end{aligned}$$

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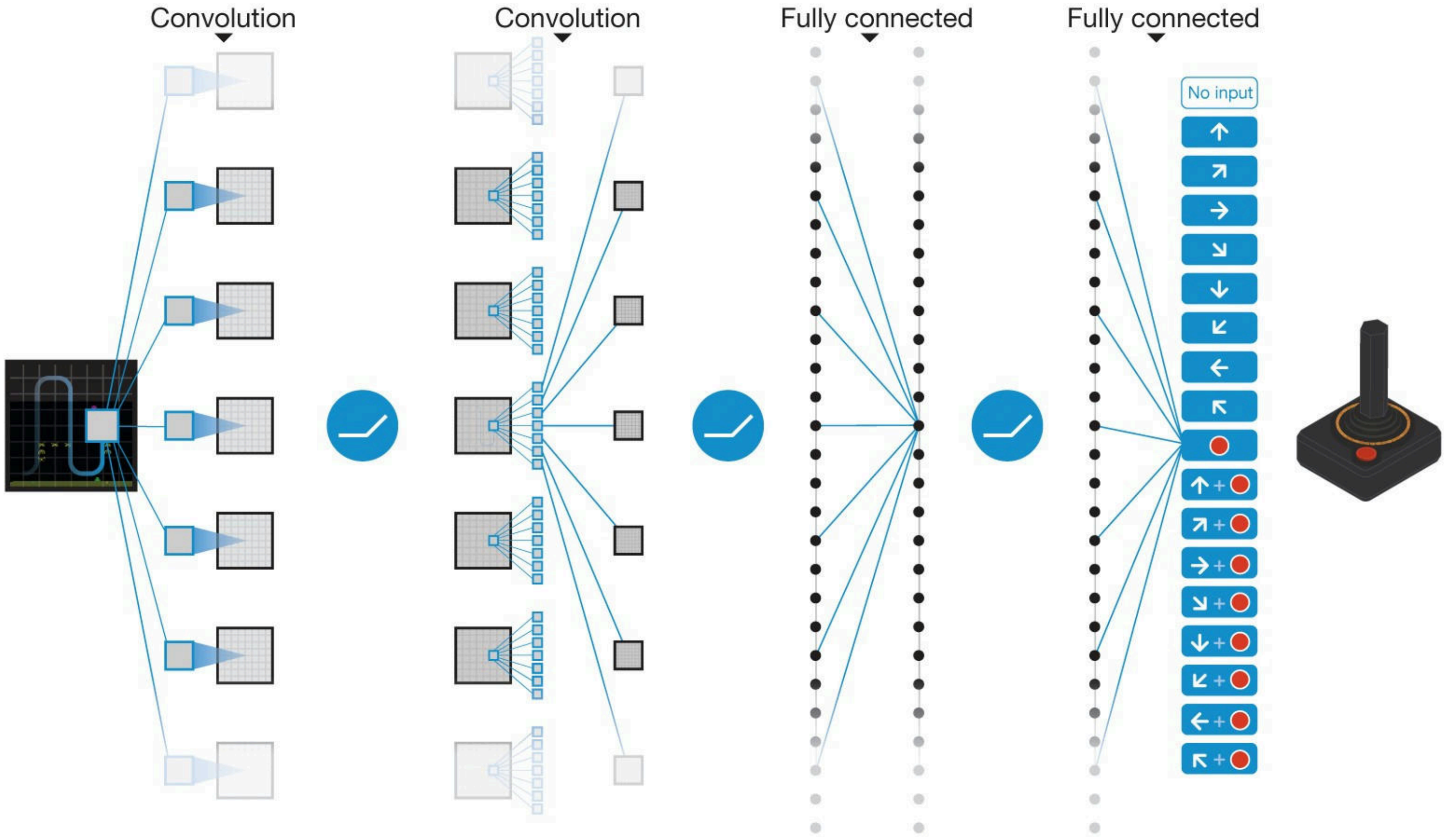

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Done with the Q function architecture, let's discuss init

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Prevents Q value overestimation (to be discussed in depth later)

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Done with init, let's discuss policy π

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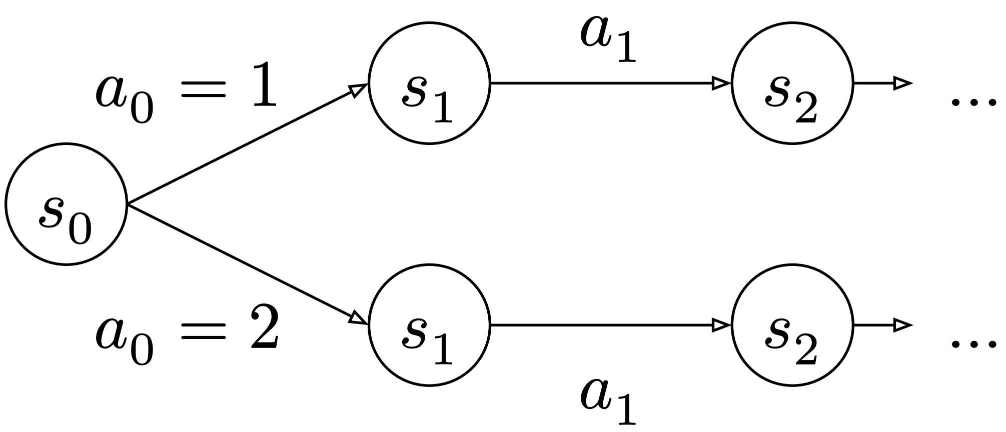

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Given these requirements, we cannot do better than random exploration

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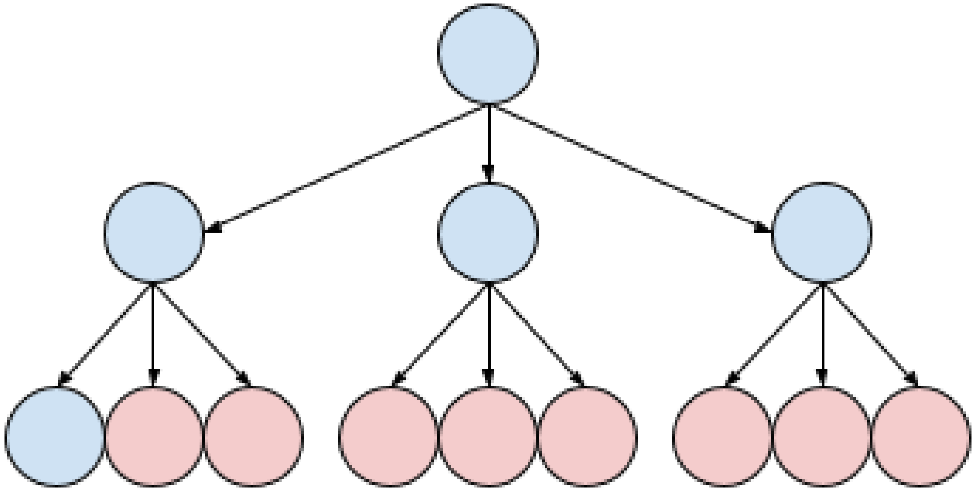

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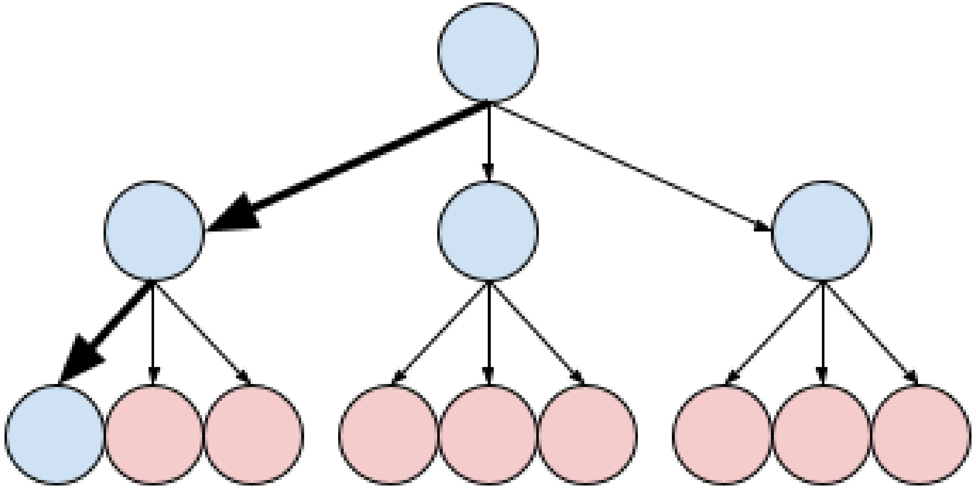

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One approach is the **ϵ-greedy** policy

$$\epsilon \in [0, 1]$$

$$u \sim \mathcal{U}[0, 1]$$

$$\pi_E(s) = \begin{cases} \mathcal{U}(A) \text{ if } u \leq \epsilon \\ \text{argmax}_{a \in A} Q(s, a) \text{ if } u > \epsilon \end{cases}$$

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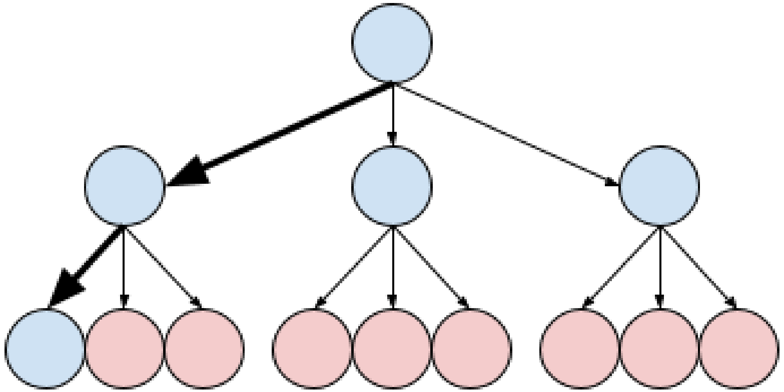
This approach is simple and works surprisingly well in practice

Exploration Policy



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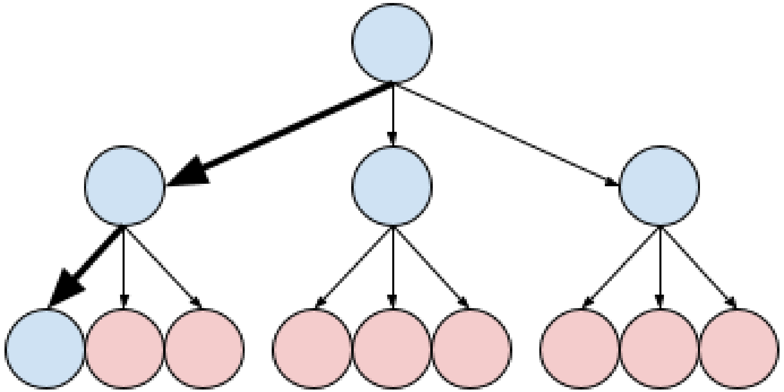

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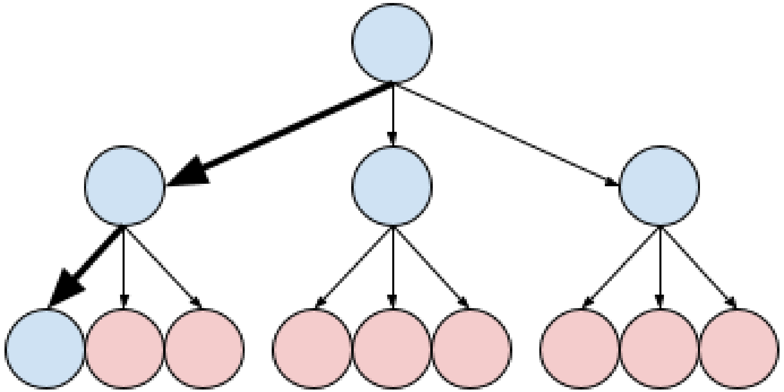

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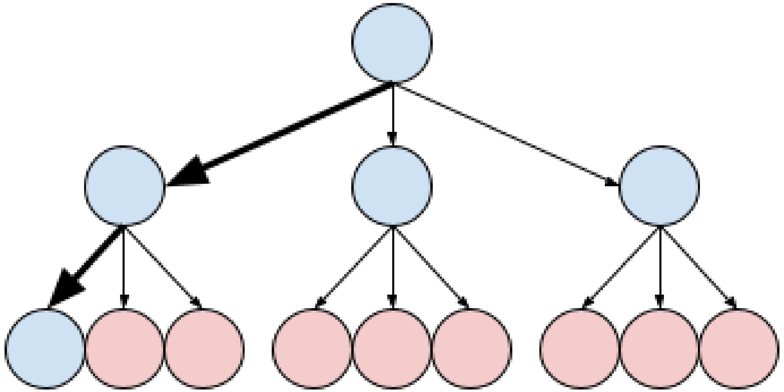

Question: Any downsides to the ε-greedy approach?

Answer: Not independently distributed. The state/action distribution will be biased by the Q function. Seems to be ignored in practice?

Exploration Policy

Summary: Maintain two policies

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π_E: A stochastic policy used for exploring the environment and collecting data

Let us make a small change to the pseudocode

```
        env = LunarLander()
        Q = nn.Module(env.state_space, env.action_space)
        theta = Q.init(seed=0)
==>     pi = policy(Q, theta)

        for update in range(num_updates):
          collected_data = collect_training_data(env, pi)
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          train_data = dataset.sample()
          theta = train(Q, theta, train_data)
          metrics = evaluate(env, pi)
```

Now we have two policies, one for collection and one for evaluation

```
        env = LunarLander()
        Q = nn.Module(env.state_space, env.action_space)
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```

Let’s move onto data collection

```
      env = LunarLander()
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Many algorithms train using a **transition tuple** (s, a, r, s', d)

Collectors

Collecting transitions correctly is deceptively tricky (off by one errors)

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```
states, next_states, rewards, actions, dones = [], [], ...
s, _ = env.reset(seed=0)
d = False
while not d:
  a = pi_e(s, theta)
  next_s, r, trunc, term, _ = env.step(action) # r = R(s')
  d = trunc or term
  states.append(s), next_states.append(next_s), rewards...
  s = next_s
# n+1 states total, but each list should be len n
episode = (states, next_states, rewards, actions, dones)
return episode
```

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We call the dataset a **replay buffer** ($\mathcal{D}$)

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Note: We often enforce a max size of $\mathcal{D}$ using a ring buffer

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We populated the dataset, now let's sample from it

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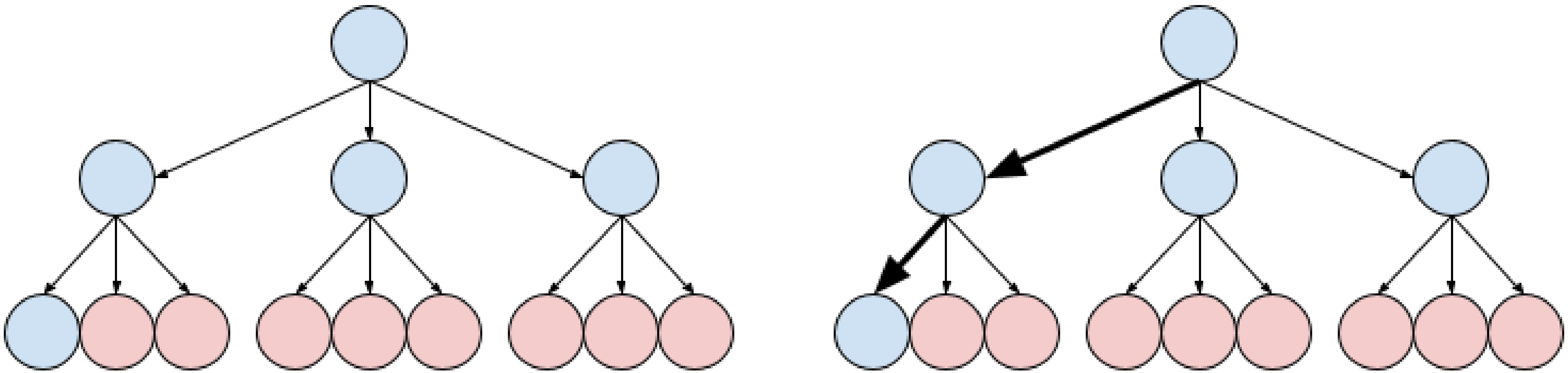

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1. **Early termination using d**
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For terminal transitions, this reduces to

$$\min_{\theta} \left(Q(s, a, \theta) - r \right)^2$$

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Traditional Q learning used a table

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Neural networks are different. Increasing a single $(s=0, a=0)$ entry will often **perturb** the Q value for all states and actions.

	$s=0$	$s=1$	$s=2$
$a=0$	**5**	**4**	**4**
$a=1$	**9**	**5**	5
$a=2$	**4**	0	0

Loss Function

These perturbations ε ripple through the Q recursion, with the max operator resulting in overestimation

$$Q(s, a, \boldsymbol{\theta}') \leftarrow r + \gamma \max_{a' \in A} [Q(s', a', \theta) + \varepsilon_\theta]$$

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Loss Function

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Initialize target parameters $\psi = \theta$

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After a while, set $\psi = \theta$ again

Loss Function

Behold, the combination of **early termination** and **target networks**

$$Q(s, a, \theta) = r + \neg \boldsymbol{d} \cdot \gamma \cdot \max_{a' \in A} Q(s', a', \boldsymbol{\psi})$$

Loss Function

Behold, the combination of **early termination** and **target networks**

$$Q(s, a, \theta) = r + \neg \boldsymbol{d} \cdot \gamma \cdot \max_{a' \in A} Q(s', a', \boldsymbol{\psi})$$

$$Q(s, a, \theta) - \left(r + \neg \boldsymbol{d} \cdot \gamma \cdot \max_{a' \in A} Q(s', a', \boldsymbol{\psi}) \right) = 0$$

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The standard objective for DQN is

$$\min_{\theta} \left(Q(s, a, \theta) - \left(r + \neg \boldsymbol{d} \cdot \gamma \cdot \max_{a' \in A} Q(s', a', \psi) \right) \right)^2$$

Loss Function

We need to make a few small updates given our new objective

```
        env = LunarLander()
        Q = nn.Module(env.state_space, env.action_space)
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          collected_data = collect_training_data(env, pi_e)
          dataset += collected_data
          train_data = dataset.sample()
==>       theta = train(Q, theta, train_data)
          metrics = evaluate(env, pi)
```

Loss Function

Initialize target parameters, and use target params in loss function

```
        env = LunarLander()
        Q = nn.Module(env.state_space, env.action_space)
==>     theta, psi = Q.init(seed=0), Q.init(seed=0)
        pi, pi_e = max_q(Q, theta), e_greedy(Q, theta)

        for update in range(num_updates):
          collected_data = collect_training_data(env, pi_e)
          dataset += collected_data
          train_data = dataset.sample()
==>       theta = train(Q, theta, psi, train_data)
          metrics = evaluate(env, pi)
```

Evaluation

Same as `collect_training_data` but use π not π_E

```
      env = LunarLander()
      Q = nn.Module(env.state_space, env.action_space)
      theta, psi = Q.init(seed=0), Q.init(seed=0)
      pi, pi_e = max_q(Q, theta), e_greedy(Q, theta)

      for update in range(num_updates):
        collected_data = collect_training_data(env, pi_e)
        dataset += collected_data
        train_data = dataset.sample()
        theta = train(Q, theta, psi, train_data)
==>     metrics = evaluate(env, pi)
```

Summary

- Review
- State of the field
- Implement Deep Q Networks (DQN) (Mnih et al.)

Next Time

- Zach, Riccardo, Grace, Dylan, and Saksham will be lecturing
  1. Give us a hint on the topic!
  2. Turn in reports (email is best)
  3. 10 min presentation + 5 min questions and discussion
     - Next lecture is not recorded (reduce anxiety)

- Miniproject handout
  - Moodle says due 22 March 16:00